A novel formulation for the Multi-Period Multi-Commodity Network Design problem with arc capacity expansion and reduction

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MONOGRAFIA SUBMETIDA AO CORPO DOCENTE DO INSTITUTO DE CIÊNCIAS EXATAS DA UNIVERSIDADE FEDERAL DE JUIZ DE FORA, COMO PARTE INTEGRANTE DOS REQUISITOS NECESSÁRIOS PARA A OBTENÇÃO DO GRAU DE BACHAREL EM CIÊNCIA DA COMPUTAÇÃO.

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A Deus, por proporcionar tantas oportunidades únicas ao longo da carreira acadêmica.

Aos meus pais e minha família, pelo apoio em todo tempo e sustento durante meus estudos.

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Resumo

O Design de Redes Multi-produto e Multi-período é uma variante interessante do problema de design de rede que planeja a estrutura de uma rede e as rotas de vários produtos ao longo de um horizonte de planejamento. Este problema tem aplicações em vários campos, tais como logística, telecomunicações, ferroviário e transporte. No entanto, uma revisão da literatura mostra que não há estudos sobre a versão do problema onde as capacidades do arcos podem mudar ao longo do horizonte de planejamento para se ajustar às demandas mutáveis dos clientes. Esta variante aumenta a complexidade do problema original através da introdução de novas decisões e tem aplicações interessantes no mundo real. Portanto, este trabalho tem como objetivo propor formulações eficazes para resolver o problema de Design de Redes Multi-produto e Multi-período. Os resultados mostram que um dos modelos MIP (do inglês Mixed Integer Programming) propostos tem uma lacuna de integralidade menor do que o outro e atinge boas soluções em tempo viável para instâncias com características distintas.

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Palavras-chave: design de redes multi-produto e multi-período; capacidades dinâmicas; programação matemática.

Abstract

The Multi-Period Multi-Commodity Network Design is an interesting variant of the network design problem, which plans the structure of a network and the routes of multiple commodities throughout a planning horizon. This problem has applications in multiple fields, such as logistics, telecommunications, railway design, and transport. However, a literature review shows that there are no studies about the dynamic variant of the problem, where arc capacities may change throughout the planning horizon to adjust to clients' demands. This variant adds extra complexity to the original problem through the introduction of new trade-offs and has interesting applications in the real-world. Therefore, this work aims to propose effective formulations to solve the Dynamic Multi-Period Multi-Commodity Network Design Problem. Results show that one of the proposed MIP models has smaller integrality gap than the other and achieves good solutions in feasible time for instances with different characteristics.

The full text is confidential and may be requested through: coord.computacao@ice.ufjf.br

Keywords: multi-period multi-commodity network design; dynamic capacities; mathematical programming.

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“Sometimes I’ll start a sentence and I don’t even know where it’s going. I just hope I find it along the way”.

Michael Scott (The Office US)

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List of Abbreviations

D-MPMCNDP	Dyanmic Multi-Period Multi-Commodity Network Design problem
GMC	General Modular Capacities
LP	Linear programming
MIP	Mixed-integer programming
MCNDP	Multi-Commodity Network Design problem
MPMCNDP	Multi-Period Multi-Commodity Network Design problem
NDP	Network Design problem
STI	Single Time-Index

1 Expanded abstract

Many real-world problems faced by companies and industries rely on optimization techniques to find the best possible solution given technical constraints. The optimal delivery route to attend multiple clients' demands for products while minimizing operational costs is an example of goal achieved by optimization algorithms. These problems are usually NP-Hard, i.e., there is not a polynomial algorithm that can find the optimal solution for any given instance. Therefore, there is a continuous interest in the literature to propose heuristic approaches, approximation algorithms, formulation tricks and improvements to find reasonably good solutions within a reasonable amount of time.

The Multi-Commodity Network Design Problem (MCNDP) is one example of problem that arises from real-world situations in fields such as logistics, railway design, transport, and telecommunications. Typically, each possible edge has construction costs, routing costs and a maximum capacity of units, and clients have demands for commodities expressed in units. The objective is to design a network capable of attending clients' demands for commodities while minimizing the costs for building the edges between clients and routing commodities. The decision process consists of determining (a) whether an arc is going to be opened and (b) how much flow of a commodity will pass through each arc. The output consists of (a) the selected subset of edges and (b) routes of each commodity throughout the network. Additional features and settings of this problem may appear depending on the practical application of the MCNDP.

Several authors have studied the MCNDP, considering different applications and proposing solving approaches that range from approximation algorithms to heuristics. However, one interesting and useful variant has yet to be more studied by the literature: the Multi-Period Multi-Commodity Network Design problem (MPMCNDP). In fact, multi-period settings for network design problems are not widely studied in the literature. The MPMCNDP takes into consideration the design of the network throughout a planning horizon, which can be seen as a discretization of time into weeks, months, years or any amount of time pertinent to the application. In addition to the complexity within

MCNDP, the demand for commodities may be different for every time period and edges may be built at the beginning of each time period to accommodate the current state of the network. Taking a planning horizon into consideration impacts directly on the final design of a network due to many new trade-offs, e.g., between building an edge earlier and paying its maintenance costs or building it later but limiting its use to later time periods.

1.1 Motivation

Even though applicable for many real-world scenarios, multi-period variants of network design problems like the MPMCNDP have been studied by only a few works in the literature. Some authors have studied the multi-period setting for network design problems focusing on other decisions rather than selection of edges. However, to the best of our knowledge, Fragkos et al. (2017) is the first work to tackle the selection of edges in a multi-period setting. They point out that maybe the high complexity of the problem, which makes it unsolvable with state-of-the-art solvers like CPLEX or Gurobi even for small-sized instances, explains the shortage of works tackling the MPMCNDP.

The variant studied by Fragkos et al. (2017) allows the opening of new edges throughout the planning horizon, but it does not allow the closure of previously opened edges. Closing edges is an interesting behaviour found in practice, either for planned maintenance or for saving costs with currently idle edges. Their model also does not allow modifications in the capacity of an edge after opening it, thus precluding either capacity expansion or reduction. However, in real-world applications, the network may change capacities following the variation of the demands. Given the applications for a variant with such behaviour, this topic becomes an interesting one for further investigation and research to meet expectations from the industry and academy.

1.2 Objective

This work aims to study a dynamic formulation for the Multi-Period Multi-Commodity Network Design Problem (D-MPMCNDP). This variant is said to be dynamic because it allows not only the opening of new edges, but also the closing, expansion, and reduction of previously opened edges throughout the planning horizon whilst considering modular capacities. To the best of our knowledge, this variant has not been studied in the literature. Thus, this study can be classified as an exploratory study of a newly proposed problem, having the primary goal of providing insights about the nature of the problem and serving as motivator for future studies in the field.

1.3 Single-period setting

The term network design is employed in the literature to describe problems with slightly distinct characteristics. Early works on the Network Design Problem (NDP), e.g., LeBlanc (1975), focus on minimizing the sum of shortest paths between all origin-destination pairs. Minimizing the overall delay of a network, i.e., the sum of weights of the shortest paths between all origin-destination pairs, is a desirable goal while designing transportation networks. However, many applications of the NDP ended up being more focused on minimizing costs and routing a given product through the network. For example, Fratta et al. (1973) mention a cost-based objective function and commodity routing constraints while discussing the application of NDP to a store-and-forward communication network.

In this context, the NDP aims to choose a subset of edges that can successfully transport commodities between nodes, fulfill each pair of origin-destination demands and minimize the cost of building the network. Since in many real-world applications the network must transport more than one commodity, the NDP expands into a Multi-Commodity Network Design problem (MCNDP). Several authors have proposed approaches to solve the MCNDP, ranging from heuristics to approximation algorithms. Gendron et al. (1999) present an initial comprehensive description of modelling decisions and solution approaches for the MCNDP. Thanks to the large number of studies in the field, many authors have presented literature reviews of the MCNDP considering specific scenarios. Recently, Yaghini and Akhavan (2012) presented a literature review of how MCNDP has been applied to solve rail freight transportation planning and Sun et al. (2015) discuss the application of the MCNDP in multi-modal networks, i.e., networks with multiple types of connections between clients according to the transportation method.

1.4 Multi-period setting

The MCNDP can be further expanded to its multi-period version, where the network state is considered for a planning horizon. Since demand for commodities may change through time, each previous demand d^k becomes d^{kt} to discriminate the relevant time period. As pointed out by Fragkos et al. (2017), there is a shortage of works in the literature which tackle the multi-period variant of the MCNDP while focusing on which and when to build edges. This section features related literature of the NDP in multi-period settings.

Commonly, multi-period variants of NDP tackle other structural aspects of the network instead of focusing on arc capacity. In this sense, for example, Kubat and Smith (2001) tackle a multi-period network design for cellular telecommunication systems. Their model decides for every time-period whether to assign a cellphone user directly to the mobile switching center or to the backbone network. However, they do not consider any adjustments in the capacity of the link between them. Later, Alumur et al. (2012) model a multi-period network design model for reverse logistics with the goal of choosing the set of inspection and re-manufacturing centers to open at a certain capacity level to maximize the revenue. However, no attention is given to choosing the capacity level of edges in the multi-period setting.

Lardeux et al. (2007) are one of the few to solve a multi-period network design problem in which capacity can change along time. However, commodity routes have to remain the same throughout the entire planning horizon and capacities can only increase. Ukkusuri and Patil (2009) propose a model for the flexible network design problem. Their goal is to maximize the flexibility of the network throughout the planing horizon. However, the authors only take into consideration the addition of capacities on arcs as potential improvements and maximize the flexibility measure, not the costs.

The work conducted by Fragkos et al. (2017) is the most recent effort to extend the network design problem to the multi-period setting whilst allowing different routes for each time period. The authors employ an arc-based formulation for the network design problem. Arcs may be installed at any time period and, once installed, they are available at all subsequent time periods.

1.5 Capacity expansion and reduction

In practice, capacities rarely expand or reduce in a continuous way. Expansion or reduction of edges happen with the installment or removal of blocks or modules. Therefore, modular capacities are commonly used in the literature to model capacity, i.e., capacities grow level by level in a discrete way. Alumur et al. (2012) is an example of work which uses modular capacities.

The use of modular capacities allows the writing of tighter formulations because variables can describe very specific transitions. Tighter formulations imply low integrality gaps, which can be seen as the distance between the LP (Linear Programming) relaxation solution and the MIP (Mixed Integer Programming) solution. The integrality gap is found by dividing the difference between the MIP solution and the LP relaxation by the MIP solution. Lower integrality gaps allow solvers to explore the branch-and-bound tree faster since many nodes can be ignored thanks to the tighter bound provided by the LP relaxation. In practice, these formulations result in finding better solutions in smaller computational time. Inspired by Jena et al. (2015), which propose tighter and more generalized formulations for the Facility Location problem thanks to the Modular Capacities, this work aims to investigate whether the use of this modeling technique is beneficial in the context of the MPCNDP.

1.6 Results

Two MIP models are proposed for the D-MPMCNDP: the Single Time Index (STI), based on the formulation presented by Fragkos et al. (2017); and the General Modular Capacities (GMC), based on the formulation presented by Jena et al. (2015). These formulations area applied in a set of adapted instances called R instances. Each instance is solved by each MIP model and its LP relaxation and the integrality gaps, solution times and overall performance are used for comparison and evaluation of each model.

In this sense, there are two main experimental questions:

Q1. How do different instance characteristics impact the solution difficulty using the two formulations?

Q2. How do the GMC and STI formulations computationally compare when solved with CPLEX?

The answer to **Q1** is that, as expected, the increase of number of nodes, arcs and commodities makes the instances more complex to solve, as well as high fixed costs and tight capacity values. The answer to question **Q2** is that, for practical use, the GMC formulation appears to be more attractive than the STI formulation because its LP relaxation is always closer to the optimal integer solution; it finds the optimal integer solution faster than STI; and it finds an integer solution for more cases than STI, even if not optimal.

1.7 Conclusion

This work studies a dynamic formulation for the Multi-Period Multi-Commodity Network Design problem, where arcs can be opened, closed, expanded or reduced over the planning horizon. Since this variant has not been seen in the literature, it proposes two MIP models for finding a solution. Results show that in practice the GMC formulation has better performance than STI thanks to the smaller integrality gap in adapted multi-period R instances. In the end, all of the initially established goals are successfully achieved.

However, there is still a lot of interesting questions related to this work. First, a more comprehensive study with different instances and different settings may bring other insights. Another related work is to analyze how the instance adaptation parameters impact the final complexity of the problem. Regarding future research work, the application of the GMC model to other network design problem variants, e.g., the problem variant where commodities can not be split, can bring interesting results for the field. Lastly, another interesting research path is to model the uncertainty in the demand using techniques such as stochastic programming instead of assuming future demand is known. In many real-world applications future demand for commodities is not known but networks still have to be designed to handle future developments.

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